PHASIC SYNTHESIS OF THE ANTENNAS WITH RANDOM PHASIC ERRORS IN THE RADIATING ARRAY

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Abstract. The problems to be solved by modern radio electronic systems become more and more complicated and diversified. This process is accompanied by the growing of different demands for such systems’ characteristics and, respectively, active searching of considerable improvement of these characteristics. There the importance of the accidental factors (errors) of amplitude-phase distribution (APD) of antenna sources on parameters and potential characteristics of transmitting antenna and system at all plays an important role.

This circumstance leads to working out and development of the statistic antenna theory – the theory of antennas with random sources.

Concerning the statistic antenna theory, many works were published. The most comprehensive one is [1].

Also are discussed the methods and algorithms of the synthesis of phase distribution needed for the operation of the transmitting apertures when the receiving antenna is situated near the Fresnel area. This case takes place in the Wireless Power Transmission (WPT) problems.

The problems of phase synthesis are complex nonlinear approximating problems that as well as for any non-linear ones, makes more difficult the elaboration of the methods of their solution. They created the problems of existence and uniqueness of the solution (i.e. required phase distribution) and also of the indicating of the iteration processes.

The following algorithm appeared to be rather effective. The iterating process was used, on each step of which the linear sub problem was decided by using the method of low squares (with usage of the pseudo inverse of matrix) for the linear phase distribution (a set of values of the phases on each sub aperture)
INTRODUCTION.

In the beginning of the paper we add same words about Wireless Power Transmission (WPT)

![Fig 1. Schedule of the electrodynamics system](image)

<table>
<thead>
<tr>
<th>Tabl.1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Communication, location, TV</strong></td>
</tr>
<tr>
<td><strong>Field on an antenna S₂</strong></td>
</tr>
<tr>
<td>Uniform</td>
</tr>
<tr>
<td>S₂ ≪ the width of Directive pattern</td>
</tr>
<tr>
<td><strong>Useful result</strong></td>
</tr>
<tr>
<td>Field value</td>
</tr>
<tr>
<td><strong>Efficiency</strong></td>
</tr>
<tr>
<td>Product of $P₁S₁$</td>
</tr>
<tr>
<td><strong>Fresnel parameter</strong></td>
</tr>
<tr>
<td>$C = \frac{ab}{\lambda d} \ll 1$</td>
</tr>
<tr>
<td><strong>Mathematical apparatus</strong></td>
</tr>
<tr>
<td>Integral equations</td>
</tr>
</tbody>
</table>
DISCONTINUOUS ANTENNA

If separate parts divide the continuous aperture and these parts (subapertures) are intuited on the radiating plate in the determinate order then the effectiveness of the WPT will be increased.

This proposition admits that the size of the focal spot is decreasing by the increasing of the whole transmitting antenna size. The intercepted portion of energy in the receiving antenna is increased when its form is fixed. But the discontinuous antenna leads to the rise of the dispersed energy (side lobes). It is felt by intuition that their some optimum must take place. The situation is based on the finding the maximum ratio of the energy flow passing through, the receiving antenna to a flow passing through whole radiating antenna. This ratio sometimes is called a coefficient of transmission or effectiveness.

This proposition at first was reported at the WPT’95" Conference Japan, Kobe, 1995. With the help of these ideas a number of practically problems of the WPT phase synthesis were solved, including the problems of creation of a uniform distribution of the field on a surface of the receiving aperture. The problems of the synthesis of a radiation field with a lower radiation level in some angular sectors or some angular directions. That is important for the construction of WPT fits the ecological requirements or the creating the best approximation to the demanded directivity pattern or the field near the Fresnel area.

The statistic theory was developing rapidly since the work [1] was published. In parallel with direct problem theory development (problems of analysis of antenna parameters with the existence of random factors), much attention was paid to reverse – statistic theory (inc. statistic antenna synthesis problems).

The attention paid to the problems of statistic antenna synthesis (first of all, of statistic phase synthesis that is very important for projecting tracing discrete transmitting systems) is in connection with the fact that in many important cases the real antenna characteristics worked out and based on the theory of determined synthesis, reasonably differ from the ones what was theoretically predicted. Such misstatement arises due to not taking into consideration the existence of accidental factors in amplitude-phase antenna sources distribution before forming the problem. Meanwhile, the high cost of modern large antennas poses the problem of increasing of their efficiency and evaluating of opportunities. It could be achieved through taking into consideration the inevitably existing accidental errors of different nature during optimization of antenna characteristics. This outlined condition explains the high practical importance of the
problems of statistic theory of problems and, respectively the rapid growth of publications about these questions in the last years.

Works [2.3] are classic. They originally described and solved the problem of the directive optimization of a common antenna array with taking into consideration the accidental factors. The work [4], describes the problems of the synthesis of antenna systems, the field set in nearest zone of radiation, Fresnele's zone. There was taken into consideration the existence of accidental errors in exciting flows distribution and errors in partial radiating fields of separate elements of discrete radiating structures. The problems of the synthesis with using the operator $M\{\ldots\}$ of statistical averaging are converted to the problems of maximizing or minimizing the functional of exciting flows' distribution. Such problems can be efficiently solved through matrix methods of optimization as an analogue to determined problems of the synthesis of WPTS [5].

The main result of the statistic antenna theory – those random errors in exciting flows distribution including the phase arrow lead to the fall of the efficiency and the growth of side leafs' level. By the way, if there's a growth of a value (dispersion) of errors, the growth of side leafs' level becomes considerable much earlier that leads to a reasonable loss in the antenna directive.

**STATISTIC PHASE SYNTHESIS**

In this paper we are concerned mostly in the affect of random errors (phase errors) on changing of efficiency – $Q$. $Q$ is determined as power radiated into stated working angle sector $u_1$ divided by total radiated power (to the angle sector $u_2$), i.e.

$$Q = \frac{\int_{u_1}^{u_2} |F(u)|^2 du}{\int_{u_1}^{u_2} |F(u)|^2 du}$$  \hspace{1cm} (1)

Here $|F(u)|^2$ is the directive pattern of antenna (DP) on power. In statistic antenna theory there the statistic averaging $M\{\ldots\}$ is used which represents the parameters of antennas as per the range of accidental factors $u$. So, the averaged effective will look like

$$P = \frac{\int_{u_1}^{u_2} M\{|F(u)|^2\} du}{\int_{u_1}^{u_2} M\{|F(u)|^2\} du}$$  \hspace{1cm} (2)

Further the works shows (formula (8), that there is a correlation at not so big errors of phasing ($< 50^\circ$):

$$M\{F|\}^2 = (1-s^2) |F(u)|^2 + s^2 \sum_{n=1}^{N} A_n^2$$  \hspace{1cm} (3)

Here $s^2$ is the phase errors disperse, $A_n$ - exciting flows amplitudes in elements of a discrete antenna. Putting (3) into (2), we get:

$$P = \frac{(1-s^2) \int_{u_1}^{u_2} |F(u)|^2 du + s^2 u_1 \sum_{n=1}^{N} A_n^2}{(1-s^2) \int_{u_1}^{u_2} |F(u)|^2 du + s^2 u_1 \sum_{n=1}^{N} A_n^2}$$  \hspace{1cm} (4)
The first item in (3) represents the decrease of a radiated field in a region of the main beam; the second item represents the appearance of additional background in a region of side radiation, i.e. its increase. As a rule $u_2 >> u_1$, so its possible to pull out the second item in a numerator (4). Then for P and Q we have:

$$\frac{P}{Q} = \frac{1-s^2}{1-s^2 + s^2 u_2 \sum_{n=1}^{N} A_n^2 / \int |F|^2 \, du}$$

From (5) we can see that its value is always below 1, i.e. the average efficiency at the existence of phase errors is always lower than in case when there are not.

So, taking into consideration the accidental mistakes leads to an original regularization of a matrix of a problem. It was mentioned works on statistic analysis of antennas.

An average Directive Pattern (DP) of phased antenna array from the aspect of power looks like:

$$M\left\{ F^2(u) \right\} = M\left\{ f^{(0)H}(\hat{\psi}) j |^2 \right\} = j^H M \left\{ f^{(0)}(\hat{\psi}) f^{(0)H}(\hat{\psi}) \right\} j = (1-\sigma^2_\psi) |F(u)|^2 + \sigma^2_\psi \sum_{n=1}^{N} A_n^2$$

and is the sum of power diagram without errors and the isotope "background". H is the sign of a transposition, $\sigma$ depending of the phasing error dispersion and $\hat{\psi}$ is a vector of angle coordinates of the energy focusing point.

Such background is proportional to a dispersion of phase errors and borders the down reachable level of side radiation. The value of this background in dB, as compared with the level of the main beam at equal-amplitude phased antenna array, can be stated in the schedule:

<table>
<thead>
<tr>
<th>Phasing discrete</th>
<th>Level of accidental background (dB)</th>
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<tbody>
<tr>
<td>$\pi/4$</td>
<td>-30,0</td>
</tr>
<tr>
<td>$\pi/8$</td>
<td>-35,5</td>
</tr>
<tr>
<td>$\pi/16$</td>
<td>-41,0</td>
</tr>
</tbody>
</table>

Fig.3 shows synthesized average DP and corresponding phase distributions at variable $\Delta\psi$ for suppressing the average level of side radiation in sector $u \in [u_1, u_2]$, $u = \sin \Theta$, of 40-elements linear equidistant equal-amplitude phased antenna array. To represent stated $F_\psi(u)$, $\delta(u)$ was taken. It guarantees the saving of the main beam of the phased antenna array at originally set syntheses excitement. The method is fully implemented for 4-5 iterations. The phased distributions looked character asymmetric. The achieved level of a side radiation decrease quite met the data of the schedule (at the decrease of main beam level for not more than 0,8 db.)
Fig. 3. Synthesis average DP and corresponding phase distribution
References